

Research Status of Algorithm Improvement for Topology Optimization of Truss Structures

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Abstract: The truss structure is a basic type of structure, which can make the best use of materials and save construction or production cost. Therefore, the research field of topology optimization of truss structure is flexible and popular. Meanwhile, the improvement of the algorithm has great research value, being helpful to push the effect and efficiency of structural design. This encourages the adaption to the complex and changeable designing conditions. Therefore, it can be better for practical uses for engineering. The research on algorithm improvement of topology optimization of truss-work is typically divided into two parts. One is aimed at the existing defects of the traditional algorithm, such as the description of real topology, large-scale advance, and foundation structure establishment. Another is methods used, not only the design parameters themselves and mechanical principles, but also mathematics tools, computer science, biology laws, and other disciplines, which can be called intelligent methods. This paper mainly expounds on the research progress of algorithm improvement for topology optimization of truss structures from the two aspects. Following that, its future research is discussed, which is about the solution of computational complexity and the development of new software.

1. Introduction

The truss structure consists of rods and hinges, all of which are subjected to axial forces. On this evidence, the performance of the material can be fully utilized, improving the utilization efficiency of the structure. This type of design product, therefore, can save the amount of material, reducing the total weight of the structure, ultimately decreasing the cost. From this looking, truss structure is popular in bridge, mechanical, and aerospace design and has a good research prospect. The application of the optimization method in structural design is structural optimization.

As is reviewed in existing research, structural optimization can be divided into three types: size optimization, shape optimization, and topology optimization (as shown in Figure 1). Dimensional optimization is aimed at adjusting small factors, for example, the size of the section or the length of the members. However, it does not change the shape of the members. This shortcoming gives birth to shape optimization, changing the shape of the boundary of the structure. Topology optimization is introduced later because the position relationship between the various components of the structure (i.e. the layout form of materials) could not be changed, which limits the degree and effect of optimization. This type of optimization is to make the design better by redistributing the positions of materials in the structure (i.e. the arrangement of node positions and the connection relations between members). Compared with size optimization and shape optimization, topology optimization has more design space and flexibility and is the most challenging part [1]. Therefore, it has the most research significance when considering the structural design field.

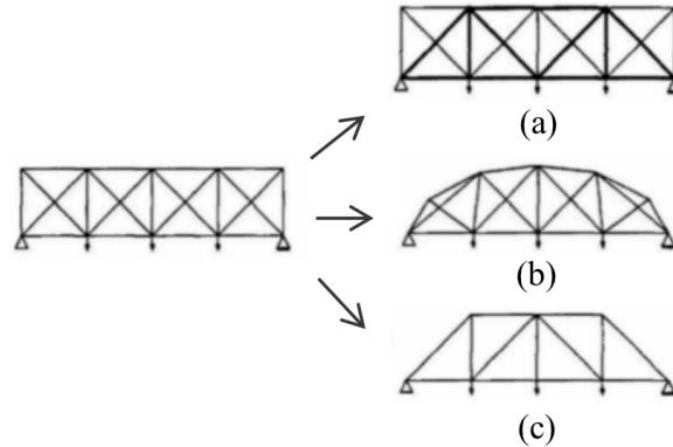


Figure 1. Structural optimization classification: (a) Size optimization; (b) Shape optimization; (c) Topology optimization [1]

The initial basic theories of topology optimization of truss structures are Michel's truss theory [2] in 1904 and Prager's classical layout theory [3] in 1977. The basic structure method proposed by Dorn et al. [4] was the real beginning research in this field and a reference for the basic ideas of structural topology optimization. The first step is to set some fixed nodes (supports) and freely moving nodes in the design area. Every two nodes can be connected with rods to form the base structure as a whole. Such rods can be added or deleted in the later optimization design [5]. For a long time, the basic idea of topology optimization is to obtain the optimal solution of the objective function by constantly adjusting the design variables based on given constraints and boundary conditions. Constraints can be divided into two types: geometric constraints, such as the web of I-section and the minimum flange thickness. Property constraints, the strength of the component, the overall stiffness of the structure, natural frequency. The objective function is the quantitative index of good or bad beam design, usually the minimum mass or volume of the structure. Design variables can typically be node coordinates, structure height, and according to whether the design variables change continuously, optimization types are separated into continuum optimization and discrete structure optimization. Almost all continuum topology optimization must be discretized first (using the finite element method), therefore the problems encountered in discrete structures also profoundly affect the progress of continuum topology optimization. The truss structure is the simplest discrete structure. Therefore, the topology optimization of truss structure is the core concept of design of discrete structure, which would be worthy of attention [6].

As the core element of optimization design, algorithm affects the efficiency of optimization design. Whether it can be successfully applied to engineering practice, it also has great research value. Since the real situation of engineering becomes more and more complex, optimization design is faced with a more challenging design environment, more complex design conditions, and more stringent design requirements. As a result, it is necessary to improve and innovate optimization design algorithms given the defects encountered by traditional methods. The general improvement trend is that the algorithm is no longer a general and fuzzy approximation. However increasingly more targeted to face special design situations, more adaptive to solve the uncertainty in optimization. Therefore, as to improve the possibility from model to practice, more in line with engineering practice.

In this article, the algorithm of topology optimization design of truss structure is studied, and the improvement of the algorithm is analyzed emphatically. Organizational structure this paper is the summary, comparison, and analysis of several main targets for algorithm improvement (traditional algorithms for the defects and corresponding improvement effects) as well as exploration of the main principles and methods adopted by the algorithm innovation. The purpose is to make readers know the status quo of algorithm improvement in the topology optimization of truss structures, which aspects are the targets of improvement, and what approaches are used to improve.

2. Targets of algorithm improvement

The initial goal of algorithm improvement is usually derived from the defects of the traditional algorithm. For example, the inapplicability of new special cases or low efficiency for more stringent optimization, determine the goal of algorithm improvement. The traditional algorithm is usually based on the fuzzy design framework, which sets the algorithm uniformly for all possible cases while ignoring the differences of various cases. Moreover, the assumption and execution must be established together with other conditions, and no one algorithm environment can control the whole optimization process. They are far from reality, only at the laboratory level. Without considering a series of large-scale practical engineering, some difficulties exist. Additionally, when designing complex problems, the more conditions and assumptions are considered, the higher the computational complexity. In addition, algorithmic implementation depends on the development of other fields such as mathematics, physics, and computers of the time. Therefore, the use of tools is limited to creating a vivid and accurate model. This part summarizes, compares, and analyzes several aspects of algorithm improvement, in other words, shortcomings of several traditional ones and corresponding improvement effects.

2.1 Description of Topology

The traditional topology optimization is based on size optimization, namely ε -method, as shown in Equation (1) [6, 7].

$$\begin{aligned} & A \in E^m \\ \min W &= \sum_{i=1}^m \rho_i A_i l_i \quad (i = 1, \dots, m) \\ \text{s.t. } & g_j(A_j) \leq 0 \quad (j = 1, \dots, J) \\ & 0 \leq A_i \leq A_i^u \text{ or} \\ & A_i \in S = \{S_1, S_2, \dots, S_N, 0\} \end{aligned} \quad (1)$$

where A is unit section design variables, and $A = (A_1, A_2, \dots, A_m)^T$; S is a discrete set of discrete variables; N is the number of elements in a discrete set; W is the structural mass; ρ_i and l_i are the density and length of the cell; g_j is the constraint function; m is pole number; J is the number of constraints; and A_i^u is the upper limit of the continuous variable A_i .

The design variable of this method is the cross-section area. If the optimization results in a zero cross-section area, the rod is removed. The operation is simple; however, the stress constraint function is not continuous, and the stress function expression is not accurate. The shape of the feasible region leads to topology optimization concave, or even being a star shape. The entire optimal solution may exist in the degenerate sub-domain of low dimension, obtaining the singular optimal solution. Therefore, it can be concluded that the traditional method has a limitation: without getting such a solution, it cannot converge to the optimal point [6, 7].

2.2. Scale Expansion

Most algorithms and innovations are kept in horizontal plane trusses on a smaller scale. Achieving a leap from plane to space is not easy, for larger-scale means more computations in terms of many aspects. First, if the optimization approaches conclude more calculations, more factors need to be added. Second, software used to deal with the method should also be improved for the application range changes accordingly. Last, when the real engineering models are introduced, many factors should be adjusted. In addition, large-scale problems are faced with rounding errors and convergence criteria. Therefore, for 3d generalization, large-scale application, and numerical problems, the stability and robustness of the algorithm need making more reasonable.

2.3. Establishment of the Basic Structure

Traditional base structures, as shown in Figure 2, have regular, rectangular design areas. In this way, the base structure of the mutually orthogonal rods is easy to operate and facilitates the optimization process. However, it does not conform to the engineering practice. The real structure area

is irregular; therefore, it might be complicated to establish the base structure. Some existing studies attempt to approach the problem via finite element network, human-computer interaction, and restricted area [9-10]. However, other difficulties are often added, such as the random placement of the endpoints of the rods, the grid nodes in the model, which makes it impossible to obtain optimal results.

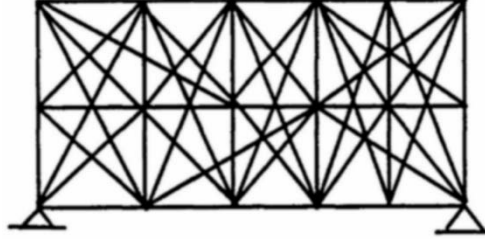


Figure 2. Basic structure diagram [8]

It can meet the irregular arbitrary shape of the design area and can be a reasonably designed node layout. Moreover, the establishment of the foundation structure can obtain the optimal solution with fewer bars, and the calculation can also be reduced [11, 12].

3. Approaches to algorithm innovation

There are many sources of algorithm innovation, mainly from scientific and technological development. To be more specific, cross-disciplinary correlation, algorithm design requirements, and mathematics are all good tools. In this paper, several common methods of algorithm improvement are introduced respectively. They are the basic elements of structural optimization design, mechanics principles, mathematical methods, and interdisciplinary intelligent methods. In this case, the three basic elements of structural optimization design are design function, constraint conditions, and objective function. It also needs to be explained that the mechanical disciplines are material mechanics, structural mechanics.

3.1. Design Elements

The improvement from design elements is based on the simple idea of the design work. Therefore, it is reliable. Mainly, function setting includes the specification of the objective function, the choice and variation characteristics of design variables, and the requirements of constraint conditions. The most common way to innovate algorithms is to start from here, which can reflect the rigorous and normative improvement process.

3.1.1. Design variables

Traditional design variables are innovated by processing the original set of data to obtain new variables to participate in the optimization design. A new parameter X is introduced into the design variables by using the method of proportion enlargement to achieve the flexibility of follow-up operations. In this case, the optimization process is simplified in advance from the point of view of design elements.

The equation of topology optimization [13] is shown as Equation (2) when the objective function is set as volume and the constraint condition is set as compliance.

$$\begin{aligned} \min_V &= \sum_{k=1}^N A_k L_k \quad (k=1, \dots, N) \\ \text{s.t. } & \mathbf{F} = \mathbf{K} \mathbf{u} \\ & \mathbf{C} = \mathbf{F}^T \mathbf{u} \leq C_l \\ & 0 < A_{\min} \leq A_k \leq A_{\max} \quad (k=1, \dots, N) \end{aligned} \quad (2)$$

The design variables is $\mathbf{A} = (A_1, A_2, \dots, A_N)^T$

where V is the total volume of truss structure; N is the number of rods; A_k is the cross-sectional area of the k rod member; L_k is the length of the k rod member; \mathbf{F} is column vector of external load; \mathbf{K} is the overall stiffness matrix of the structure. \mathbf{u} is the column vector of node displacement; $\mathbf{C} = \mathbf{F}^T \mathbf{u}$ is the

structural flexibility, reflecting the overall stiffness of the truss structure under the current load; C_l is the allowable value of structural compliance; A_{\min} is the lower limit set to avoid singularity of stiffness matrix; A_{\max} is the maximum allowable cross-sectional area of member.

Using the maximum allowable cross-sectional area of the rod, the design variables are processed as follows:

$$x_k = A_k / A_{\max}$$

Then, the optimized model of Equation (2) is transformed into Equation (3):

$$\begin{aligned} \min_x V &= \sum_{k=1}^N x_k L_k A_{\max} \quad (k=1, \dots, N) \\ \text{s.t. } \mathbf{F} &= \mathbf{K} \mathbf{u} \\ \mathbf{C} &= \mathbf{F}^T \mathbf{u} \leq C_l \end{aligned} \quad (3)$$

$$0 < A_{\min} / A_{\max} \leq x_k \leq 1 \quad (k=1, \dots, N)$$

New design variables are obtained: $\mathbf{x} = (x_1, x_2, \dots, x_N)$

3.1.2. Constraint Condition

The objective function is designed to minimize the structural strain energy. Based on the energy principle and the principle of full stress, and through verification, the same optimization results can be achieved without constraint conditions and volume. This type of algorithm does not need to design volume constraints, which is a great breakthrough. The principle is as follows:

Relationship between structural strain energy and volume is shown as Equation (4):

$$C(A) = \sum_{i=1}^n \frac{F_i^2 L_i}{2EA_i} = \frac{1}{2E} \sum_{i=1}^n \sigma_i^2 A_i L_i \quad (4)$$

Since the optimal structural member is in the state of full stress, assume that the stress of the member is, then the equation is improved is as Equation (5):

$$C(A) = \sigma_0^2 \sum_{i=1}^n A_i L_i = \frac{\sigma_0^2}{2E} V \quad (5)$$

In this case, the structural strain energy is proportional to the volume, which objective function is, the optimization results are the same [14].

3.1.3. Objective Function

By introducing the node cost into the objective function of the problem, the number of overlapping rods in the optimal topology can be reduced to a certain extent, which is of great significance to the actual engineering structure design. The node cost introduces the objective function, to reduce the overlapping number of rods, which is of great significance to the actual construction difficulty and cost control. The node cost is introduced into the objective function. It has a lot to do with what should be considered in engineering practice [6].

3.2. Mechanical Principles

Using mechanical principles to improve, the best way to reflect the function of the structure itself, is also the most convincing way to optimize. Structure optimization design as the purpose of the bearing, first of all, should give full play to the theory of mechanics, material mechanics, structural mechanics and mechanics of elasticity. Starting from the underlying logic, it can ensure that the key tasks of structure can be done well. It provides a reliable basis to the development of new algorithm.

3.2.1. Principal Stress Trace Method

The principal stress trace method in tensile and compression model of reinforced concrete structure is applied. The main tensile stress trace guides the arrangement of the structure of the reinforced concrete beam. The tensile stress is borne by the steel bars, and the bearing capacity of the concrete is increased.

The topological design of the optimal truss structure is determined according to the optimal force transmission path. The idea is to improve both the bearing capacity and stiffness of the structure, and to reduce its volume, because that they are interrelated in the mechanics of structures and materials.

The drawing step is to use the equivalent statics knowledge, using software such as ANSYS and COMSOL, determine the magnitude of the principal stress direction, then start drawing.

This method is based on mechanical principles and plots suitable for the first and third principal stress traces. According to the generalized Hooke's law of anisotropic material structure, the trace lines of principal stress and principal strain of anisotropic material structure coincide, using this conclusion to complete the improvement of optimization algorithm. Shown as Equation (6).

$$\begin{bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{zz} \\ \alpha_{xy} \\ \alpha_{yz} \\ \alpha_{zx} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} & \alpha_{26} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \alpha_{35} & \alpha_{36} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \alpha_{45} & \alpha_{46} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} & \alpha_{55} & \alpha_{56} \\ \alpha_{61} & \alpha_{62} & \alpha_{63} & \alpha_{64} & \alpha_{65} & \alpha_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix} \quad (6)$$

The component of shear stress in the direction of principal stress, as Equation (7) :

$$\begin{bmatrix} \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} \alpha_{41} & \alpha_{42} & \alpha_{43} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \\ \alpha_{61} & \alpha_{62} & \alpha_{63} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{bmatrix} \quad (7)$$

If $\tau_{xy}=\tau_{yz}=\tau_{zx}=0$, then in the equation, the conditions for non-zero solutions of $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$ are as Equation (8):

$$\begin{vmatrix} \alpha_{41} & \alpha_{42} & \alpha_{43} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \\ \alpha_{61} & \alpha_{62} & \alpha_{63} \end{vmatrix} = 0 \quad (8)$$

The intersection of the first and the third principal stress traces is beneficial to the arrangement of joints and rods, thus improving the topology optimization [15].

3.2.2. Design Criteria for Full Stress

Fully Stressed Design (FSD) is based on the basic knowledge of theoretical mechanics and mechanics of materials, the cross section area of each rod is adjusted therefore that the internal force of each rod reaches the allowable stress. In this way, the strength of the material can be Fully utilized and the final optimization result is the minimum structural weight. When only one load combination is acted on the truss, there is only one working condition. If there is more than one working condition on the truss, the full stress design requires that each rod be in full stress state in at least one working condition. For statically determinate truss, since the internal force of the rod depends only on the equilibrium condition and has nothing to do with the cross-section of the rod, the internal force can be calculated first, and then the cross-section of the rod can be determined as by the allowable stress of the material [16-18].

3.3. Mathematical Methods

Mathematical methods are a good medium for providing rigorous inferential support and computational modelling tools for scientific problems. No matter the first one is encountered, starting from the basic elements of the model (optimization function, design variables) or mechanics principles, mathematical methods have unique advantages, and mathematics is also a good medium from topology optimization algorithms to other fields of solutions and mutual reference. With the support of mathematics, the progress of optimization can be greatly accelerated and the space to play is also larger. It is not limited to the optimization design itself and structural mechanics itself, but the whole scientific field.

3.3.1. Least-Square Method

The least-square method is an important statistical model. In essence, it calculates the minimum of the sum of squares of errors and constructs the function to approximately replace the original set of data, which is a branch of mathematics. It typically appears in data fitting, image processing, and other operations [19, 20]. The linear least square method was used for smoothing processing to help topology optimization, mainly for its fitting function. The smoothing equation is in Table 1 [21], scatters the

non-smooth curve in the plane coordinate system, and the approximate straight line is obtained. In this way, the rough cross-section contour of the topology optimization result can be obtained, which is also convenient for much subsequent processing, such as modelling the reconstruction of the symmetric optimization result. Suppose that in the plane rectangular coordinate system, there are i scatter fields and scatter points in each scatter, then as in Table 1:

Table.1. Approximate linear parameter [21]

The horizontal coordinate of the scatter	x_i^l
The y-coordinate of the scatter	y_i^l
Total number of scatter points:	n^l
Straight slope	$a^l = \frac{n^l \sum_{i=1}^n x_i^l y_i^l - \sum_{i=1}^n x_i^l}{n^l \sum_{i=1}^n (x_i^l)^2 - (\sum_{i=1}^n x_i^l)^2}$
Linear intercept	$b^l = \frac{\sum_{i=1}^n (x_i^l)^2 - \sum_{i=1}^n x_i^l \sum_{i=1}^n x_i^l y_i^l}{n^l \sum_{i=1}^n (x_i^l)^2 - (\sum_{i=1}^n x_i^l)^2}$
The smoothed line	$y=a^l x+b^l$

Where $i=1,2,3\cdots, n$; $l=1,2,3\cdots, m$.

3.3.2. Fuzzy Dynamic Penalty Function

The method is to input variable \mathbf{r}_1 of the algorithm, and the proportion of infeasible individuals is small (\mathbf{S}_1), the proportion of infeasible individuals is large (\mathbf{L}_1). It is used to cover the theory domain $[0, 1]$ of input variable \mathbf{r}_1 . The Triangle membership function is selected for a fuzzy subset, and its distribution is shown in Figure 3 (a).

The distribution of language variables and membership function of input variable \mathbf{r}_2 is the same as that of input variable \mathbf{r}_1 , which is represented by a small proportion of infeasible individuals (\mathbf{S}_2), the medium proportion of infeasible individuals (\mathbf{M}_2), and a large proportion of infeasible individuals (\mathbf{L}_2), respectively.

Output variable \mathbf{C} and the penalty factor is small (\mathbf{S}_v). Small penalty coefficient (\mathbf{S}), and medium penalty coefficient (\mathbf{M}), the large penalty factor (\mathbf{L}), the penalty factor is large (\mathbf{L}_v), used to cover the domain of output variable \mathbf{C} $[0.001, 1]$. Then the triangle membership function is selected for a fuzzy subset, and its distribution is as shown in Figure 3 (b).

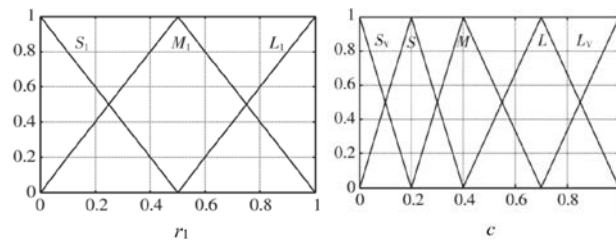


Figure 3. (a) Fuzzy subset distribution of penalty coefficient c ; (b) Fuzzy subset distribution of infeasible individual ratio r_1 [17]

3.4. Intelligence

In addition to mechanics and mathematics, several types of science are making continuous progress. Due to the application of computer programs, scientific principles can be introduced into the algorithm improvement of topology optimization design. They approach the problems that the assumptions cannot be modified, and the conditions cannot be satisfied or predicted. This approach is characterized by advanced, flexible and accurate methods, such as breaking away from traditional infrastructure methods and relying on fixed ideas. At the same time, other cross-disciplines can be used to simplify the calculation, reducing the computational complexity.

3.4.1. Graphic Collision Detection Technology

It belongs to virtual simulation technology and is applied to computer game animation [22]. It is effective for judging whether the rods intersect. It can specifically analyze the if, when, and where problems of two objects colliding, by establishing Boolean analysis, and analyzing the time and contact mode of the collision. The algorithm can cover line segments, rectangles, cuboids, polygons, cylinders, spheres, and planes. Take line segment and circle intersection test as an example, as shown in Figure 4.

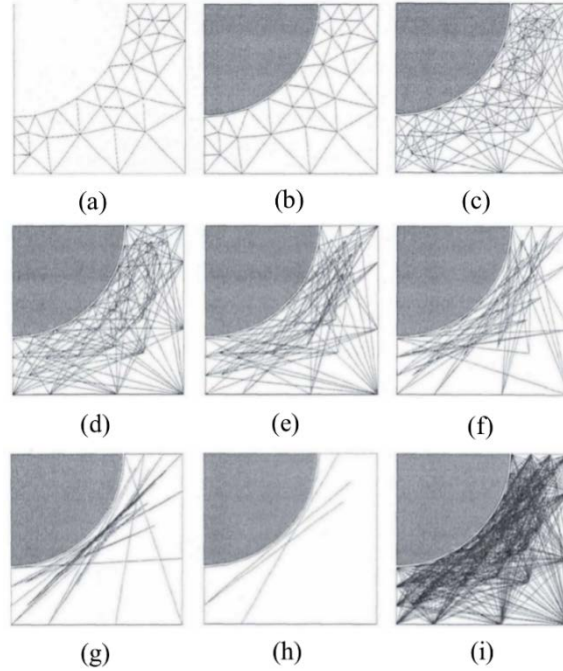


Figure 4. Line segment and circle intersection test (a) Meshing; (b) Lv1 base structure; (c) Lv2 base structure; (d) Lv3 base structure; (e) Lv4 base structure; (f) Lv5 base structure; (g) Lv6 base structure; (h) Lv7 by base structure; (i) Fully connected base structure [22]

3.4.2. Natural Organism

3.4.2.1. Genetic Algorithm

Genetic algorithm (GA), which originated in the 1960s, is a random optimization method that mimics biological evolution in nature. It is based on genetic theory and natural selection to explain the random exchange of individual chromosome information in a population. It can deal with multiple models, multiple objectives, and non-linearity, and has good robustness. In addition, automatic control and image processing can be controlled well. The application scope includes not only structural topology optimization but also composite materials [23] and post-disaster road traffic recovery [24]. Moreover, the method itself is also constantly improved [25]. The most advanced improvement at present is segmenting inheritance, in which the test function is a non-negligible element. Represented by the Rastrigin function, as shown in Equation (9), the relationship between the convergence of algorithms with crossover probability P_c and mutation probability P_m under different settings is explained. This function has multiple local minima, however only one global minimum is at the point (0, 0), and the global minimum is 0. The function image is shown as Figure 5.

$$f(x, y) = 20 + x^2 + y^2 - 10(\cos 2\pi x + \cos 2\pi y) - 5 \leq x, y \leq 5 \quad (9)$$

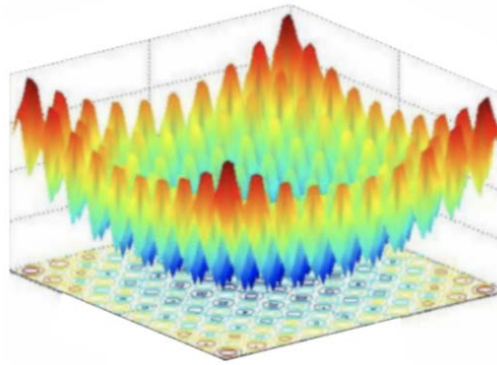


Figure 5. Rastrigin function [26]

3.4.2.2. Wolves Algorithm

This is a relatively advanced algorithm, first proposed in 2011. The description algorithm program is vivid and intelligent according to the living habits of wolves [27]. Wolves have the division of labor, food distribution, and they are divided into different roles and identities [28]. Such an approach would combine the characteristics, the wisdom of nature and engineering to achieve uniform principles and improve efficiency. Although this approach requires long-term observation of wolves, the rationality of algorithm programs does not need to be worried because it is tested in nature for a long time. Attention should only be paid to how to apply the program to the rationalization process of topology optimization [6].

4. Conclusion

Topological optimization of truss structures, especially the improvement of algorithms, is a meaningful and progressive research topic. Each element of the field is a relatively cutting-edge part and much progress has been achieved over the past years. This paper reviewed the key targets and the main methods of the improvement of algorithms of the topological optimization of truss structures. It can be seen that the objective of algorithm improvement is the defects of the traditional algorithm and the problems in the optimization process. For instance, targeted improvement measures were taken to solve the main defects of traditional modelling, description, and scale expansion. At the same time, to achieve a better innovation effect and to promote the improvement of optimization effect and efficiency, the improvement method begins with design elements. To be more flexible, from the laboratory to practical engineering applications, and to simplify calculations rather than add complexity, it is necessary to investigate much other scientific knowledge. Math and mechanics are useful, on the other hand, computers and even biological evolution and animal behaviour have strong advantages. In the future, there is still a lot of room for innovation. It depends on other interdisciplinary and technological development to lead the progress of ages. Algorithm improvement of complex processes must be accompanied by the enhancement of computational complexity and operability of software and model. In other words, the progress of software development can realize the improvement effect of the algorithm. Otherwise, the improvement of the algorithm can only stay at the level of formula, not serving the goal of topology optimization.

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